

On the Relationship between Financial Instability and Economic Performance: Stressing the Business of Nonlinear Modeling*

David Ubilava[†]

School of Economics, University of Sydney

E-Mail: david.ubilava@sydney.edu.au

Abstract

The recent global financial crisis and the subsequent recession have revitalized the discussion on causal interactions between financial and economic sectors. In this study, I apply the financial stress and the national activity indices – respectively developed by Federal Reserve Banks of Kansas City and Chicago – to investigate the impact of financial uncertainty on an overall economic performance. I examine nonlinear dynamics in a vector smooth transition autoregressive framework, and illustrate regime-dependent asymmetries in the financial and economic indices using the generalized impulse–response functions. The results reveal more amplified dynamics during the stressed conditions. I further evaluate benefits of nonlinear modeling in an out-of-sample setting. The forecasting exercise brings out the important advantages that nonlinear modeling provides in the identification of the causal effect of financial instability on overall economic performance.

Keywords: business cycles; financial stress; forecast evaluation; vector STAR models.

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1 Introduction

The most recent global financial crisis and the subsequent recession have fostered further discussion of the scope and the extent of causal links between financial and economic sectors (e.g., [Gertler and Kiyotaki, 2010](#); [Jermann and Quadrini, 2012](#); [Brunnermeier and Sannikov, 2014](#); [Schleer and Semmler, 2015](#)). The topic is not new, however. Over eighty years ago, [Schumpeter \(1934\)](#) pointed to the benefits of financial services for economic growth, while [Fisher \(1933\)](#) and [Keynes \(1936\)](#) attributed recessions in part to financial market failures (for further discussion, see [King and Levine, 1993](#); [Brunnermeier and Sannikov, 2014](#)). In the intervening years, the question of whether financial development results in economic growth, or, alternatively, financial crises cause economic downturns, has been broadly discussed and analyzed in the economics literature (see, for example, [King and Levine, 1993](#); [De Gregorio and Guidotti, 1995](#); [Levine, 1997](#); [Arestis et al., 2001](#); [Calderón and Liu, 2003](#); [Gilchrist et al., 2009](#)). The rationale behind the postulated relationship has been widely accepted. What has been much more disputed, however, is the scope of financial variables to forecast economic variables – i.e., the degree of out-of-sample [Granger](#) causality between the economic and financial variables. After all, as [Samuelson \(1966\)](#) put it: “Wall Street indices predicted nine out of the last five recessions!”

Business cycles, as defined by [Burns and Mitchell \(1946\)](#), reflect the co-movement of multiple individual economic time series ([Diebold and Rudebusch, 1996](#)). A logical way to analyze cycles thus consists of examining the dynamics of a composite index of economic variables (see [Stock and Watson, 1989, 1999a](#)). Such indices have long been used as leading indicators for economic expansions and recessions (e.g., [Stock and Watson, 1993](#); [Granger and Teräsvirta, 1993](#); [Lahiri and Wang, 1994](#); [Stock and Watson, 2003](#); [Camacho, 2004](#); [Marcellino et al., 2006](#); [Anderson et al., 2007](#)), albeit with varying degrees of success (see, for example, [Diebold and Rudebusch, 1991](#); [Giacomini and Rossi, 2013](#)). Factor methods

have been applied to financial variables as well, to obtain a composite index, better known as the financial stress index (e.g., [Hakkio and Keeton, 2009](#); [Hubrich and Tetlow, 2015](#)).

The main focus of this study is to investigate the relationship between financial and economic indices in a forecasting environment. Much like previous studies (such as [Davig and Hakkio, 2010](#); [Liu et al., 2011](#); [Mittnik and Semmler, 2013](#); [Brunnermeier and Sannikov, 2014](#); [Hubrich and Tetlow, 2015](#); [Jones and Enders, 2016](#)), this research also relies on the assumption that regime-dependent nonlinear models are better suited for analyzing linkages between financial and economic series. There exist a number of grounds for expecting such an “episodic” character in the relationship, and this episodic nature may be one of the reasons researchers in the past have failed to find much evidence of financial friction affecting economic activity (see [Hubrich and Tetlow, 2015](#)). First of all, economic agents tend to behave differently in times of higher financial uncertainty, as compared to a relatively stable environment ([Hubrich and Tetlow, 2015](#)). [Davig and Hakkio \(2010\)](#) offer a rationale for this discrepancy in terms of financial accelerator theories. Furthermore, [Schleer and Semmler \(2015\)](#) suggest that the interest rate on borrowing is low and remains constant in low stress regime, but becomes a nonlinear function of the leverage ratio in high stress regime. These considerations suggest that the effect of financial stress on economic activity may be nonlinear and will vary, depending on the state of the financial sector. Whether or not allowing for this nonlinearity produces more accurate forecasts is the question to be investigated here.

This study contributes to, and complements, the existing literature in two main areas. First, it applies a vector smooth transition modeling framework, with the financial stress index used as the transition variable – a specification that introduces an additional nonlinear link to the dynamic relationship between the two sectors. Notably, [Schleer and Semmler \(2015\)](#) used a similar methodology to address regime-dependent behavior of the financial and economic sectors in the euro area, and more recently, [Jones and Enders \(2016\)](#) applied the univariate version of the methodology to the U.S. data. Second, and in contrast to any

previous study, the current paper proceeds a step further by assessing the forecast accuracy of the models under consideration, as the approach lends itself naturally to the out-of-sample [Granger](#) causality testing (e.g., [Ashley et al., 1980](#)).

The findings reveal asymmetric dynamics, of the kind that would be expected under regime-dependent behavior. In addition, and more importantly, improved forecasting performance is found to arise from vector smooth transition autoregressive modeling. The results of this study add new insights to those available in the existing and growing body of literature that has addressed the linkages between financial and economic sectors in light of the recent financial crisis.

2 Econometric Modeling and Forecasting

This study adopts the regime-dependent modeling framework to examine nonlinearities in the financial stress and economic activity indicators. Smooth transition regressions were first proposed by [Bacon and Watts \(1971\)](#), and subsequently advocated in a time series context by [Chan and Tong \(1986\)](#). More formally, [Luukkonen et al. \(1988\)](#) and [Teräsvirta \(1994\)](#) introduced and developed smooth transition autoregressive (STAR) modeling and testing frameworks. The family of smooth transition models was later extended to the multivariate framework (e.g., [Anderson and Vahid, 1998](#); [Rothman et al., 2001](#)), resulting in the vector smooth transition autoregressive (VSTAR) model (e.g., [Teräsvirta and Yang, 2014a](#)).

The VSTAR framework facilitates the multivariate analysis in a manner that allows for cases in which the extent of causal relationship varies across regimes, depending on the transition variable's state. In this modeling framework, a switch between the regimes is allowed to occur either instantaneously or “smoothly,” with the latter case featuring the continuum of intermediate steps between the two regimes. A smooth transition process is appealing from a theoretical standpoint, as it accounts for the aggregation effect across

heterogeneous economic agents (Jones and Enders, 2016). Since their introduction, the smooth transition models have gained popularity and have been given consideration in studies that model asymmetric cyclical variations of business cycles, and other related economic indicators (e.g., Teräsvirta and Anderson, 1992; Teräsvirta, 1995; van Dijk and Franses, 1999; Rothman et al., 2001; Skalin and Teräsvirta, 2002; Franses and van Dijk, 2005; Schleer and Semmler, 2015). The following section briefly outlines the modeling and testing frameworks of STAR-type models, with an emphasis on the multivariate case. The reader is referred to Hubrich and Teräsvirta (2013), Teräsvirta and Yang (2014a), and Teräsvirta and Yang (2014b) for a more in-depth review and details.

2.1 Univariate and Multivariate Smooth Transition Models

Consider a basic *linear* autoregressive model of order p , $AR(p)$:

$$y_t = \alpha + \sum_{i=1}^p \beta_i y_{t-i} + \varepsilon_t \quad (1)$$

where y_t is the dependent variable in period t ; α and β_i , $i = 1, \dots, p$, are parameters defining the dynamic convergence properties of the model; and $\varepsilon_t \sim iid(0, \sigma_\varepsilon^2)$ is a white noise process.

The linearity assumption might be relaxed in a variety of ways. But this paper applies the smooth transition *nonlinear* modeling framework of Teräsvirta (1994) for this purpose. The STAR model introduces a particular type of regime-dependency in the autoregression:

$$y_t = \alpha_0 + \sum_{i=1}^p \beta_{0i} y_{t-i} + \left[\alpha_1 + \sum_{i=1}^p \beta_{1i} y_{t-i} \right] G(s_t; \gamma, c) + \varepsilon_t \quad (2)$$

where $G(s_t; \gamma, c)$ is the so called transition function. This function's values are bounded between zero and one, and can take any value over that range – with the actual value attained depending on the transition variable, s_t , and the smoothness and location parameters, γ and

c . γ determines the speed at which changes in regimes occur, while c determines the value within the range of s_t , on which regime changes are centered.¹

There are several choices for the transition functions, the most popular of which is the logistic function given by:

$$G(s_t; \gamma, \mathbf{c}) = \left\{ 1 + \exp \left[-\gamma^* \prod_{j=1}^k (s_t - c_j) \right] \right\}^{-1} \quad \gamma^* > 0, \quad c_1 < \dots < c_k \quad (3)$$

where $\mathbf{c} = (c_1, \dots, c_k)$ is a vector of centrality parameters. In practice most analysts choose either $k = 1$ or $k = 2$ – choices that result in logistic STAR (LSTAR) and quadratic STAR (QSTAR) models, respectively. The former is useful in situations where asymmetries in autoregressive dynamic in relation to s_t are suspected; the latter is useful for situations where nonlinearity in dynamics is linked to the absolute value of s_t , with an implied assumption that $c_1 = -c_2$ in (3). Finally, in equation (3) the parameter γ is normalized by σ_s^k , $\gamma^* = (\gamma/\sigma_s^k)$, where σ_s is the sample standard deviation of the transition variable. This normalization has the effect of making the smoothness parameter unit-free.

The smooth transition autoregressive framework can be extended to a multivariate setting (e.g., Rothman et al., 2001; Camacho, 2004). To begin, consider a linear vector autoregressive model of order p , VAR(p):

$$\mathbf{y}_t = \boldsymbol{\alpha} + \sum_{i=1}^p \mathbf{B}_i \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t \quad (4)$$

where $\mathbf{y}_t = (y_{1t}, \dots, y_{nt})'$ is an n -dimensional vector of dependent variables; $\boldsymbol{\alpha}$ and \mathbf{B}_i , $i = 1, \dots, p$, are n -dimensional vector and matrices of parameters; and $\boldsymbol{\varepsilon}_t \sim iid(\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon)$, where $\boldsymbol{\Sigma}_\varepsilon$ is a positive definite covariance matrix.

A nonlinear version of equation (4), the vector smooth transition autoregressive model

¹A special case of this functional form is the threshold autoregressive (TAR) model pioneered by Tong and Lim (1980). For this special case, the transition function becomes, in essence, an indicator function.

of order p , $VSTAR(p)$, is specified as follows:

$$\mathbf{y}_t = \boldsymbol{\alpha}_0 + \sum_{i=1}^p \mathbf{B}_{0i} \mathbf{y}_{t-i} + \boldsymbol{\Gamma}_t \left[\boldsymbol{\alpha}_1 + \sum_{i=1}^p \mathbf{B}_{1i} \mathbf{y}_{t-i} \right] + \boldsymbol{\varepsilon}_t \quad (5)$$

where $\boldsymbol{\Gamma}_t = \text{diag}\{G(s_{1t}; \gamma_1, \mathbf{c}_1), \dots, G(s_{nt}; \gamma_n, \mathbf{c}_n)\}$ is a diagonal matrix of smooth transition functions. The aforementioned specification implies that the transition functions (possibly) differ across the equations. Alternatively, the transition functions may be restricted to be the same across the equations, i.e., $\boldsymbol{\Gamma}_t = G(s_t; \gamma, \mathbf{c}) \mathbf{I}_n$, where \mathbf{I}_n is the n -dimensional identity matrix. A middle ground between these extremes would correspond to the case in which the transition variable is common across the equations, but the parameters – and thus the transition functions – differ.

2.2 Testing Linearity and the Adequacy of STAR-type Models

A crucial component in the building of STAR-type models is the testing for linearity. This step should precede estimation of any nonlinear model, because one cannot *directly* test the hypothesis of linearity, that is the null hypothesis of $\gamma = 0$, in a STAR (or VSTAR) model. The unavailability of the option of a direct test arises from the problem associated with unidentified nuisance parameters, also known as the [Davies' problem](#) ([Davies, 1977, 1987](#)). To illustrate the point, consider equation (2) in conjunction with equation (3): the nonlinear model will reduce to the linear $AR(p)$ either by imposing $\gamma = 0$ or by imposing $\alpha_1 = \beta_{11} = \dots = \beta_{1p} = 0$. To circumvent the issue, [Luukkonen et al. \(1988\)](#) proposed to approximate the transition function using third-order Taylor series expansion around $\gamma = 0$. This results in a testable auxiliary regression:

$$y_t = \phi_0 + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{j=1}^3 \sum_{i=1}^p \psi_{ji} y_{t-i} s_t^j + \nu_t \quad (6)$$

where ϕ_0 , ϕ_i , and ψ_{ji} , $i = 1, \dots, p$, $j = 0, \dots, 3$, are parameters of the auxiliary regression, while ν_t combines the original error term, ε_t , and the approximation error resulting from the Taylor series expansion. Under the null, $\nu_t \equiv \varepsilon_t$, and an LM-type test (which only requires estimation of the model under the null) can be used to test for linearity against the STAR alternative. The linearity test is equivalent to a joint test of $H'_0: \psi_{ji} = 0, \forall j, i$. The F -test has desirable size properties in small and moderate samples (e.g., [Teräsvirta et al., 2010](#)).

A similar approach can be applied to test linearity in a multivariate setting ([Camacho, 2004](#); [Teräsvirta and Yang, 2014a](#)). In particular, the system of auxiliary regressions can be given by:

$$\mathbf{y}_t = \phi_0 + \sum_{i=1}^p \Phi_i \mathbf{y}_{t-i} + \sum_{j=1}^3 \mathbf{S}_t^j \left[\sum_{i=1}^p \Psi_{ji} \mathbf{y}_{t-i} \right] + \nu_t \quad (7)$$

where ϕ_0 , Φ_i , and Ψ_{ji} , $i = 1, \dots, p$, $j = 0, \dots, 3$, are parameter vectors and matrices of the auxiliary regression, and where, assuming a common transition function across the equations, $\mathbf{S}_t = s_t \mathbf{I}_n$; $\nu_t \sim iid(\mathbf{0}, \Sigma_\nu)$ is a vector of error terms, combining the original and the approximation errors. In small samples, the higher-order polynomial terms can be ignored ([Weise, 1999](#); [Teräsvirta and Yang, 2014a](#)), yielding a somewhat more “economical” version of the auxiliary regression:

$$\mathbf{y}_t = \phi_0 + \sum_{i=1}^p \Phi_i \mathbf{y}_{t-i} + \mathbf{S}_t \left[\sum_{i=1}^p \Psi_{1i} \mathbf{y}_{t-i} \right] + \nu_t \quad (8)$$

The linearity test is equivalent to a system-wide test of $\Psi_{ji} = \mathbf{0}, \forall j, i$. Rao’s F -test has desirable size properties in small and moderate samples ([Teräsvirta and Yang, 2014a](#)).

Once the VSTAR (or STAR, in the univariate case) have been estimated, an important final step is to conduct an array of misspecification tests. These include tests for no remaining (additive) nonlinearity; remaining parameter constancy; and no remaining serial correlation. The testing framework is similar to the aforementioned, as it is carried out in an auxiliary

regression setting. See [Teräsvirta \(1994\)](#); [Teräsvirta and Yang \(2014a\)](#) for details concerning univariate and multivariate nonlinear models, respectively.

2.3 Testing Forecast Accuracy and Granger Causality

An improved in-sample fit of less parsimonious models, say VAR or VSTAR as compared to a simple AR, does not necessarily guarantee improved out-of-sample predictability. The latter is a testable hypothesis. Consider a one-step-ahead point forecast, given by:

$$\hat{y}_{t+1|t} = f(\mathcal{F}_t; \hat{\theta}_t), \quad (9)$$

where $f(\cdot)$ is the functional form of the estimated model, and $\hat{\theta}_t$ is a set of parameter estimates; \mathcal{F}_t is the information set available at the time of forecast. The associated forecast error is given by:

$$\hat{e}_{t+1} = y_{t+1} - \hat{y}_{t+1|t} \quad (10)$$

where y_{t+1} is the actual realization of the variable of interest in the forecast period. Forecast accuracy is assessed based on a loss function, $L(\hat{e}_{t+1})$, which can take various different forms. Under the assumptions of a quadratic loss, i.e., $L(\hat{e}_{t+1}) = \hat{e}_{t+1}^2$, the forecast accuracy can be evaluated using the mean-square forecast error (MSFE) measure:

$$\hat{\delta}^2 = \frac{1}{P} \sum_{t=R}^{T-1} \hat{e}_{t+1}^2 \quad (11)$$

where P is the total number of out-of-sample forecasts, and R denotes the estimation window size, i.e., the subset of observations used to obtain a set of parameter estimates, so that $R + P = T - 1$, where T denotes the total sample size. The foregoing outlines the pseudo-forecasting exercise. In the course of this exercise, the available sample is split into

the estimation and the forecasting sub-samples, and a relatively large number of forecasts are obtained to be evaluated for accuracy.

Consider forecasts from two competing models. The models can differ due to composition of the information set, as well as the functional form. If, for example, one forecast is obtained based on the information set that omits a potentially causal variable, while the information set of the other forecast incorporates the potentially causal variable, then the test of equal forecast accuracy is effectively the test of Granger non-causality in a forecasting environment. This lines up closely with the original notion of Granger-causality testing (e.g., [Ashley et al., 1980](#)). In the current context, let $\mathcal{F}_t^{(1)} = \{y_t, y_{t-1}, \dots\}$, and $\mathcal{F}_t^{(2)} = \{y_t, y_{t-1}, \dots, z_t, z_{t-1}, \dots\}$, where $\{z_t\}$ is the potentially causal variable. Then $\hat{\delta}_1^2 > \hat{\delta}_2^2$, or equivalently $\hat{\delta}_1^2 - \hat{\delta}_2^2 > 0$, would imply that $\{z_t\}$ Granger causes $\{y_t\}$, where $\hat{\delta}_i^2$, $i = 1, 2$, are the mean squared forecast errors from the two competing models.

One of the more frequently applied metrics in the assessment of forecast accuracy is the [Diebold and Mariano \(1995\)](#) (DM) statistic, which tests predictive accuracy of two (non-nested) competing models. But when one of the competing models nests another (which is certainly the case in Granger causality testing), the standard asymptotic critical values are no longer applicable (e.g., [Clark and McCracken, 2001](#)). For such cases, [McCracken's](#) critical values can be applied ([McCracken, 2007](#)), despite the lack of clear guidance in cases in which the competing models are nonlinear (e.g., [Ferrara et al., 2015](#)). Alternatively, the test can be modified as per [Clark and West \(2007\)](#), which incorporates an adjustment factor in the forecast accuracy statistic. Let $\hat{e}_{1,t+1}$ and $\hat{e}_{2,t+1}$ be the forecast errors from two competing models, where the latter model nests the former. Then the null hypothesis of equal mean squared (forecast) errors, or currently Granger non-causality, is rejected if the difference given by $\hat{\delta}_1^2 - (\hat{\delta}_2^2 - \hat{\delta}_a^2)$ is sufficiently positive, where $\hat{\delta}_a^2$ is the sample variance of an “adjustment error” given by $\hat{e}_{a,t+1} = \hat{y}_{t+1|t}^{(1)} - \hat{y}_{t+1|t}^{(2)}$, where $\hat{y}_{t+1|t}^{(1)}$ and $\hat{y}_{t+1|t}^{(2)}$ are one-step-ahead forecasts from the restricted and unrestricted models ([Clark and West, 2007](#)).

The regression-based test of equal forecast accuracy is equivalent to a test of $H_0 : \mu = 0$, against $H_a : \mu > 0$, in the following regression setting:

$$\ell_{t+1} = \mu + \xi_{t+1}, \quad t = R, \dots, T - 1 \tag{12}$$

where, in obtaining the DM statistic, $\ell_{t+1} = \hat{e}_{1,t+1}^2 - \hat{e}_{2,t+1}^2$ is regressed on a constant, and the sample t -values are compared with [McCracken](#)'s critical values; while in obtaining the CW statistic, $\ell_{t+1} = \hat{e}_{1,t+1}^2 - \hat{e}_{2,t+1}^2 + \hat{e}_{a,t+1}^2$ is regressed on a constant, and the sample t -values are compared with standard normal critical values. To account for possible autocorrelation in the results, the heteroskedasticity and autocorrelation consistent (HAC) standard errors are applied.

3 Empirical Framework

This research uses monthly series of the financial stress index (FSI) of the Federal Reserve Bank of Kansas City, and the national activity index (NAI) of the Federal Reserve Bank of Chicago, as proxies for financial instability and economic performance, respectively. The data span the period from February 1990 to December 2014. The respective indices are constructed based on principal components methodology, and incorporate an array of major financial and economic indicators. In particular, the FSI is based on 11 financial variables, containing credit and liquidity spreads, and measures of asset price behavior ([Hakkio and Keeton, 2009](#)). The behavior of the index lines up well with much other evidence on the timing of financial instability episodes during the sample period. In addition, the index is highly correlated with other indices designed for similar purposes (for example, the financial stress index applied by [Hubrich and Tetlow \(2015\)](#), as well as indices tabulated by the Federal Reserve Banks of St Louis, Chicago, and Cleveland; see also [Hakkio and Keeton \(2009\)](#) for a

brief review of other available indices of financial stress).² The NAI is based on 85 economic variables, and corresponds to the economic activity index of [Stock and Watson \(1999b\)](#). Key benefits associated with using this index are that it is a monthly measure (as opposed to the quarterly GDP measures), and, moreover, it combines an array of economic factors, and does not focus on a single aspect of an economy ([Hakkio and Keeton, 2009](#)). [Figure 1](#) brings out the strong negative correlation between the two indices – the financial stress episodes coincide with decreased economic activity, and the fluctuations in both indices accurately convey the fact of a severe financial crisis and recession in the late 2000s.

[[Figure 1](#) here]

3.1 Model Specification, Estimation, and Evaluation

The initial stage of the modeling procedure involves specification of an econometric model to be estimated. This stage identifies the linear specification of the univariate and multivariate models, and tests linearity against the respective STAR-type alternatives.

First, unit root hypotheses are examined using Augmented Dickey–Fuller (ADF) test statistics. The test results (available upon request) suggest that both time series are weakly stationary. Therefore, the levels of these indices are used in the analysis.

Next, the bivariate vector autoregressive model is specified. The autoregressive lag length, p , is set to four, based on the system-wide Bayesian Information Criterion (BIC), and subject to the requirement that there be no residual autocorrelation. Likewise, the (restricted) univariate model is specified as an AR(4) process.

Once the linear models are identified, the linearity tests are carried out using the procedure described in the previous section. Lags of the financial stress index, i.e., z_{t-d} , $d = 1, \dots, p$, are considered as candidate transition variables. Test results reject the linearity

²[Schleer and Semmler \(2015\)](#) applied similar measures of financial stress in the euro area.

assumption in favor of the logistic VSTAR model, with z_{t-1} yielding the lowest probability value (see Table 1 for details). Therefore, z_{t-1} is identified as a suitable transition variable.

[Table 1 here]

The next stage involves model estimation. In nonlinear modeling, starting values of the parameters matter. The estimation stage thus consists of several steps. Initially, starting values are selected on an equation-by-equation basis, using a gridsearch procedure. The gridsearch, in essence, isolates the optimal set of parameters, conditional on a range of predetermined values of the smoothness and location parameters, that minimize the sum of squared residuals of a given equation. These parameters are then applied as starting values in a univariate nonlinear least squares estimation. First, the smooth transition models are fit separately for each equation of the VSTAR model. Then, the parameter estimates from the previous step are incorporated as starting values to estimate the VSTAR model in a seemingly unrelated regression setting. The estimation relies on the Gauss-Newton optimization algorithm.

The parameter estimates for the linear VAR and nonlinear VSTAR models are given in Table 2. The estimated transition function associated with the VSTAR model follows:³

$$G(s_t; \hat{\gamma}, \hat{c}) = \left\{ 1 + \exp \left[\frac{-11.24/\sigma_z}{(4.95)} \left(z_{t-1} - \frac{0.97}{(-)} \right) \right] \right\}^{-1} \quad (13)$$

Several features of the transition function should be noted. First, the transition function is restricted to be the same across the equations.⁴ Second, in the process of estimation, the location parameter, c , converged to a value in excess of the 90th percentile of the transition

³In addition, the univariate STAR model also yielded qualitatively similar parameter estimates, which, for the sake of brevity, are not presented here.

⁴A somewhat more flexible version would imply that the smoothness and location parameters, and thus the transition functions, are not restricted to be the same across the equations. A preliminary analysis suggested that there were hardly any benefits (or qualitative differences) in such specification, as compared to the currently specified model. Hence, the VSTAR with a common transition function was maintained.

variable – the predetermined upper bound. Therefore, instead of using the estimate of c , I set the value of the location parameter equal to the upper bound of the transition variable. This restriction ensures that sufficient number of observations are retained in the “stressed” regime for the purpose of parameter identification. As the value of the location parameter was fixed rather than estimated, there is no standard error associated with it. Finally, the transition function appears to be “moderately” smooth, as illustrated in Figure 2. Thus, one set of parameters defines the model dynamics during the normal regime, and a different set of parameters – during the stressed regime (see Table 2); and the switch between the regimes occurs gradually, and is centered on approximately a positive one standard deviation of the financial stress index.

[Table 2 here]

[Figure 2 here]

The final stage of the modeling procedure consists of a diagnostic evaluation of the estimated nonlinear models. Probability values associated with the tests of no remaining nonlinearity, of parameter constancy, and of no serial correlation are presented in Table 1. As is evident from the table, the tests fail to reject any of these hypotheses, suggesting no apparent misspecification issues with the estimated two–regime logistic VSTAR model. To better illustrate the dynamics of this model, I now turn to the impulse–response analysis.

3.2 Interpretation: Generalized Impulse–Response Analysis

A special treatment is needed when generating impulse–responses from nonlinear models, because their dynamics depend on the information set prior to the shocks, i.e., the histories; the sign and the magnitude of the shocks; and the idiosyncratic disturbances, i.e., the noise that occurs throughout the forecast horizon. It follows that so called naïve extrapolation

will yield biased results and is not valid here. In the face of this problem, [Koop et al. \(1996\)](#) proposed a numerical approximation technique that produces the generalized impulse–response (GIR) functions. A GIR for a horizon h , in the case of a given shock, $v_t \in \Upsilon_t$, and history, $\omega_{t-1} \in \Omega_{t-1}$, is defined as a realization of a random variable:

$$\text{GIR}_Y(h, \Upsilon_t, \Omega_{t-1}) = E(Y_{t+h} | \Upsilon_t, \Omega_{t-1}) - E(Y_{t+h} | \Omega_{t-1}), \quad h = 0, 1, \dots \quad (14)$$

where ω_{t-1} denotes a point in time with initial conditions from the subset of histories under consideration, and v_t denotes the realization of an initial shock.

The goal of the exercise is to illustrate asymmetries in the dynamic responses to financial and economic shocks. This paper follows a bootstrap resampling algorithm, similar to that of [Skalin and Teräsvirta \(2002\)](#), extended to a multivariate setting (e.g., [Weise, 1999](#)). First, the overall model dynamics are illustrated using 100 randomly sampled histories (without replacement) from all 288 available histories. For each sampled history, 100 pairs of initial shocks – i.e., shocks from the same period, associated with each of the considered two equation – are randomly sampled (with replacement) from the vectors of residuals of the estimated VSTAR model. These shocks are sampled on the basis of the residuals that are greater (less) than one positive (negative) residual standard deviation. In this way, four distinct scenarios are generated. For each history–shock combination, 500 extrapolates of length equal to 24 (i.e., two years) are generated with and without the initial shock, and using vectors of residuals that are randomly sampled (with replacement) from the estimated VSTAR model. The extrapolates are then averaged across all bootstrap iterations within each history–shock combination. A difference of these averages yields the GIR associated with a particular history and initial shock. Given 100 considered histories, and 100 initial shocks per history, a total of 10,000 GIRs are obtained for each scenario considered (i.e., with positive and negative economic and financial shocks). They are illustrated in [Figure 3](#)

using the highest density region plots of [Hyndman \(1995, 1996\)](#).

[Figure 3 here]

Several features are apparent. First, the impulse–responses tend to converge to a spike, which is indicative of stability of the estimated VSTAR model. Second, there is little evidence of economic shocks manifesting in economically meaningful and statistically significant responses in the financial sector; the financial shocks, however, appear to have some, albeit marginal, effect on the economic activity. Third, the sign–specific asymmetries are apparent, particularly when examining the NAI responses to FSI shocks: on average, a negative shock in the financial sector results in a better economic performance, but the opposite does not necessarily appear to be true. In either case, the effects appear to be modest.

The foregoing exercise might be camouflaging a potentially strong causal relationship between the financial and economic variables in stressed conditions. That is, history–specific asymmetries is a likely feature of the nonlinear model (see also [Jones and Enders, 2016](#)). To consider this possibility, generated are GIRs for histories coinciding with the periods when the financial stress index is greater than unity and, simultaneously, the national activity index is less than negative unity (recall, the indices are constructed so that their standard deviations are equal to one). These histories evidently represent the stressed regime. Alternatively, the normal regime is defined when the absolute values of both indices are less than 0.5 (see [Figure 4](#)). The stressed regime contains a total of 18 histories, and all of them are used in the impulse–response analysis. For purposes of comparison, another set of 18 histories are randomly sampled from the pool of histories associated with the normal regime. The rest of the procedure is the same as before. The results of this exercise are presented in [Figures 5 and 6](#).

[Figure 4 here]

[Figure 5 here]

[Figure 6 here]

Several additional features emerge from these figures. First, there is clear evidence of history-specific asymmetries: the GIRs are more amplified during the stressed conditions, as compared with their values under the normal regime. Of particular interest are responses to economic shocks. Although the financial stress index is essentially non-responsive to shocks during normal times, the picture changes dramatically in the stressed conditions, when GIRs of the financial stress index are greatly amplified during the first year following the shock (top-right panel). This effect is partly due to more amplified responses to own shocks in the national activity index (top-left panel). Second, the previously noted sign-specific asymmetries are more apparent here. This is particularly true in the case of dynamic responses to FSI shocks; in this case, there are bimodal densities of GIRs at intermediate horizons. The regime-dependent nonlinear dynamics is also brought out clearly here, as we observe the regime switch from stressed to normal conditions, associated with a dramatic improvement in economic activity.

The asymmetries revealed by the generalized impulse-response analysis testify to the existence of clear nonlinear dynamics in the system of financial stress and national activity indices. The results accord with other related studies that apply similar financial and economic variables in a nonlinear setting (e.g., [Davig and Hakkio, 2010](#); [Mittnik and Semmler, 2013](#); [Hubrich and Tetlow, 2015](#)). In particular, there is an evidence of more amplified positive responses in economic performance after negative financial shocks during stressed conditions. This is in agreement with the findings of [Mittnik and Semmler \(2013\)](#); [Schleer and Semmler \(2015\)](#), suggesting that indeed, there is a good chance that regulation policies designed to reduce financial stress might be particularly effective during stressed conditions. In turn, the financial sector is responsive to economic shocks during stressed conditions – a phenomenon that is absent during normal conditions, however. This finding is also consistent with [Schleer and Semmler \(2015\)](#), who find that for a majority of considered European

countries, nonlinear modeling establishes causality from economic to financial sectors, which is camouflaged in a linear setting. These results illustrate the adequacy of nonlinear modeling. To see whether this also translates to an improved out-of-sample fit, I now turn to a forecasting exercise.

3.3 Forecast Evaluation and Granger Causality

This paper uses a rolling window approach to obtain P one-step-ahead forecasts. The first estimation window starts in January 1991 and ends in December of 2009. Every successive window is rolled over by one month, resulting in 59 one-step-ahead forecasts from each considered model. The forecasting exercise is intentionally designed so that every estimation window contains the high-financial-stress/low-economic-activity period of 2008-2009. The linearity test is carried out in each rolling window, using the first four lags of the financial stress index as candidate transition variables. In all instances, the first lag of the financial stress index turned out to be the suitable transition variable. Four models are estimated in each rolling window: the linear autoregressive and vector autoregressive models, i.e., equations (1) and (4); the vector smooth transition autoregressive model, i.e., equation (5); and the smooth transition autoregressive model, i.e., equation (2). Note that the national activity index is modeled as some function of the financial stress index in all these models, except for the linear autoregression. Moreover, because the univariate smooth transition autoregressive model incorporates the financial stress index, the latter is implicitly assumed to be weakly exogenous. The aim is to maintain some features of the VSTAR model, but to fit a more “parsimonious” model, one featuring a simplified representation of the channel through which the financial stress index can affect the dynamics of the national activity index. Under this procedure, the one-step-ahead forecasts are obtained based on the information set and the parameter estimates of all four models.

The results of this forecasting exercise are presented in Table 3. The table features the

absolute and relative measures of root mean squared forecast errors (RMSFE) from each considered model. The forecast accuracy measures suggest improved predictability of the national activity index when the financial stress index is incorporated into the regression, be that in a linear or nonlinear manner. That is, there is strong out-of-sample evidence that the financial stress index does Granger cause the national activity index. Of all models considered, the VSTAR performs the best. In particular, it statistically significantly improves forecast accuracy relative to STAR and VAR models. The finding leads to the main takeaway of this study: the financial stress index facilitates improved forecasting of economic activity, and nonlinearity matters in identifying such causal relationship.

[Table 3 here]

4 Conclusion

There has been a growing interest in examining the relationship between financial and economic variables, particularly in a nonlinear framework. Nonlinearity has long been recognized as a characteristic feature of business cycles (e.g., [Teräsvirta and Anderson, 1992](#)), and more recently a series of studies have adopted various nonlinear modeling techniques to better interpret the nontrivial dynamics in financial and economic sectors ([Davig and Hakkio, 2010](#); [Mittnik and Semmler, 2013](#); [Hubrich and Tetlow, 2015](#); [Schleer and Semmler, 2015](#); [Jones and Enders, 2016](#)). This study contributes to the existing literature by applying vector smooth transition autoregressive framework to the U.S. financial stress and national activity indices, and more importantly, by extending the nonlinear model evaluation in a forecasting environment.

The findings of this research, by and large, accord with and complement those of the aforementioned studies. The specific results here suggest that dynamics vary considerably across the normal and the volatile regimes: both financial and economic variables are more

responsive to shocks during the stressed conditions compared to the normal regime. From the forecasting standpoint, the findings of this exercise support the hypothesis that financial stress Granger-causes economic performance, and the effect is most apparent when the nonlinearity in the system of equations is accounted for.

An out-of-sample forecast comparison was performed for the period after the most recent recession. This was done in order to address identification, that is, to obtain a sufficiently large amount of observations in the volatile regime. The downside of this procedure is that much of the forecast accuracy measures are assessed in the normal regime (i.e., the stable period following the global financial crisis). Even so, the benefits of nonlinear modeling are clearly evident, as a more proper allowance for the nonlinear dynamics that are present in the data, has led to more accurate forecasts.

References

- Anderson, H. M., G. Athanasopoulos, and F. Vahid (2007). Nonlinear Autoregressive Leading Indicator Models of Output in G-7 Countries. *Journal of Applied Econometrics* 22(1), 63–87.
- Anderson, H. M. and F. Vahid (1998). Testing Multiple Equation Systems for Common Nonlinear Components. *Journal of Econometrics* 84(1), 1–36.
- Arestis, P., P. O. Demetriades, and K. B. Luintel (2001). Financial Development and Economic Growth: The Role of Stock Markets. *Journal of Money Credit and Banking* 33(1), 16–41.
- Ashley, R., C. W. Granger, and R. Schmalensee (1980). Advertising and Aggregate Consumption: An Analysis of Causality. *Econometrica* 48(5), 1149–1167.
- Bacon, D. and D. Watts (1971). Estimating the Transition between Two Intersecting Straight Lines. *Biometrika* 58(3), 525–534.
- Brunnermeier, M. K. and Y. Sannikov (2014). A Macroeconomic Model with a Financial Sector. *American Economic Review* 104(2), 379–421.
- Burns, A. F. and W. C. Mitchell (1946). *Measuring Business Cycles*. National Bureau of Economic Research.
- Calderón, C. and L. Liu (2003). The Direction of Causality between Financial Development and Economic Growth. *Journal of Development Economics* 72(1), 321–334.
- Camacho, M. (2004). Vector Smooth Transition Regression Models for US GDP and the Composite Index of Leading Indicators. *Journal of Forecasting* 23(3), 173–196.
- Chan, K. and H. Tong (1986). On Estimating Thresholds in Autoregressive Models. *Journal of Time Series Analysis* 7(3), 179–190.
- Clark, T. and M. McCracken (2001). Tests of Equal Forecast Accuracy and Encompassing for Nested Models. *Journal of Econometrics* 105(1), 85–110.
- Clark, T. and K. West (2007). Approximately Normal Tests for Equal Predictive Accuracy in Nested Models. *Journal of Econometrics* 138(1), 291–311.
- Davies, R. (1977). Hypothesis Testing when a Nuisance Parameter is Present only under the Alternative. *Biometrika* 64(2), 247–254.
- Davies, R. (1987). Hypothesis Testing when a Nuisance Parameter is Present only under the Alternative. *Biometrika* 74(1), 33–43.
- Davig, T. and C. S. Hakkio (2010). What Is the Effect of Financial Stress on Economic Activity? *Federal Reserve Bank of Kansas City, Economic Review* 95(2), 35–62.

- De Gregorio, J. and P. E. Guidotti (1995). Financial Development and Economic Growth. *World Development* 23(3), 433–448.
- Diebold, F. and R. Mariano (1995). Comparing Predictive Accuracy. *Journal of Business & Economic Statistics* 13(3), 253–263.
- Diebold, F. X. and G. D. Rudebusch (1991). Forecasting Output with the Composite Leading Index: A Real-Time Analysis. *Journal of the American Statistical Association* 86(415), 603–610.
- Diebold, F. X. and G. D. Rudebusch (1996). Measuring Business Cycles: A Modern Perspective. *Review of Economics and Statistics* 78(1), 67–77.
- Ferrara, L., M. Marcellino, and M. Mogliani (2015). Macroeconomic Forecasting During the Great Recession: The Return of Non-linearity? *International Journal of Forecasting* 31(3), 664–679.
- Fisher, I. (1933). The Debt-Deflation Theory of Great Depressions. *Econometrica* 1(4), 337–357.
- Franses, P. H. and D. van Dijk (2005). The Forecasting Performance of Various Models for Seasonality and Nonlinearity for Quarterly Industrial Production. *International Journal of Forecasting* 21(1), 87–102.
- Gertler, M. and N. Kiyotaki (2010). Financial Intermediation and Credit Policy in Business Cycle Analysis. In B. M. Friedman and M. Woodford (Eds.), *Handbook of Monetary Economics*, Volume 3, pp. 547–599. Elsevier.
- Giacomini, R. and B. Rossi (2013). Forecasting in Macroeconomics. In N. Hashimzade and M. A. Thornton (Eds.), *Handbook of Research Methods and Applications on Empirical Macroeconomics*. Edward Elgar Publishing.
- Gilchrist, S., V. Yankov, and E. Zakrajšek (2009). Credit Market Shocks and Economic Fluctuations: Evidence from Corporate Bond and Stock Markets. *Journal of Monetary Economics* 56(4), 471–493.
- Granger, C. and T. Teräsvirta (1993). *Modelling Nonlinear Economic Relationships*. Oxford University Press, USA.
- Granger, C. W. (1969). Investigating Causal Relations by Econometric Models and Cross-Spectral Methods. *Econometrica* 37(3), 424–438.
- Hakkio, C. S. and W. R. Keeton (2009). Financial Stress: What Is It, How Can It Be Measured, and Why Does It Matter? *Federal Reserve Bank of Kansas City, Economic Review* 94(2), 5–50.

- Hubrich, K. and T. Teräsvirta (2013). Thresholds and Smooth Transitions in Vector Autoregressive Models. Research Paper 13, CREATES.
- Hubrich, K. and R. Tetlow (2015). Financial Stress and Economic Dynamics: The Transmission of Crises. *Journal of Monetary Economics* 70, 100–115.
- Hyndman, R. J. (1995). Highest-Density Forecast Regions for Nonlinear and Non-normal Time Series Models. *Journal of Forecasting* 14(5), 431–441.
- Hyndman, R. J. (1996). Computing and Graphing Highest Density Regions. *American Statistician* 50(2), 120–126.
- Jermann, U. and V. Quadrini (2012). Macroeconomic Effects of Financial Shocks. *American Economic Review* 102(1), 238–271.
- Jones, P. M. and W. Enders (2016). The Asymmetric Effects of Uncertainty on Macroeconomic Activity. *Macroeconomic Dynamics forthcoming*, 1–28.
- Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*. Macmillan.
- King, R. G. and R. Levine (1993). Finance and Growth: Schumpeter Might Be Right. *Quarterly Journal of Economics* 108(3), 717–737.
- Koop, G., M. Pesaran, and S. Potter (1996). Impulse Response Analysis in Nonlinear Multivariate Models. *Journal of Econometrics* 74(1), 119–147.
- Lahiri, K. and J. G. Wang (1994). Predicting Cyclical Turning Points with Leading Index in a Markov Switching Model. *Journal of Forecasting* 13(3), 245–263.
- Levine, R. (1997). Financial Development and Economic Growth: Views and Agenda. *Journal of Economic Literature* 35(2), 688–726.
- Liu, Z., D. F. Waggoner, and T. Zha (2011). Sources of Macroeconomic Fluctuations: A Regime-Switching DSGE Approach. *Quantitative Economics* 2(2), 251–301.
- Luukkonen, R., P. Saikkonen, and T. Teräsvirta (1988). Testing Linearity Against Smooth Transition Autoregressive Models. *Biometrika* 75(3), 491–499.
- Marcellino, M., J. H. Stock, and M. W. Watson (2006). A Comparison of Direct and Iterated Multistep AR Methods for Forecasting Macroeconomic Time Series. *Journal of Econometrics* 135(1), 499–526.
- McCracken, M. W. (2007). Asymptotics for Out of Sample Tests of Granger Causality. *Journal of Econometrics* 140(2), 719 – 752.
- Mittnik, S. and W. Semmler (2013). The Real Consequences of Financial Stress. *Journal of Economic Dynamics and Control* 37(8), 1479–1499.

- Rothman, P., D. van Dijk, and P. H. Franses (2001). Multivariate STAR Analysis of Money–Output Relationship. *Macroeconomic Dynamics* 5(4), 506–532.
- Samuelson, P. A. (1966). Science and Stocks. *Newsweek* September 19, 92.
- Schleer, F. and W. Semmler (2015). Financial Sector and Output Dynamics in the Euro Area: Non-linearities Reconsidered. *Journal of Macroeconomics* 46, 235 – 263.
- Schumpeter, J. A. (1934). *The Theory of Economic Development: An Inquiry into Profits, Capital, Credit, Interest, and the Business Cycle*. Harvard University Press. Translated by Redvers Opie.
- Skalin, J. and T. Teräsvirta (2002). Modeling Asymmetries and Moving Equilibria in Unemployment Rates. *Macroeconomic Dynamics* 6(2), 202–241.
- Stock, J. H. and M. W. Watson (1989). New Indexes of Coincident and Leading Economic Indicators. *NBER Macroeconomics Annual* 4, 351–409.
- Stock, J. H. and M. W. Watson (1993). A Procedure for Predicting Recessions with Leading Indicators: Econometric Issues and Recent Experience. In J. H. Stock and M. W. Watson (Eds.), *Business Cycles, Indicators and Forecasting*, pp. 95–156. University of Chicago Press.
- Stock, J. H. and M. W. Watson (1999a). Business Cycle Fluctuations in US Macroeconomic Time Series. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Volume 1, pp. 3–64. Elsevier.
- Stock, J. H. and M. W. Watson (1999b). Forecasting Inflation. *Journal of Monetary Economics* 44(2), 293–335.
- Stock, J. H. and M. W. Watson (2003). Forecasting Output and Inflation: The Role of Asset Prices. *Journal of Economic Literature* 41(3), 788–829.
- Teräsvirta, T. (1994). Specification, Estimation, and Evaluation of Smooth Transition Autoregressive Models. *Journal of the American Statistical Association* 89(425), 208–218.
- Teräsvirta, T. (1995). Modelling Nonlinearity in US Gross National Product 1889–1987. *Empirical Economics* 20(4), 577–597.
- Teräsvirta, T. and H. Anderson (1992). Characterizing Nonlinearities in Business Cycles using Smooth Transition Autoregressive Models. *Journal of Applied Econometrics* 7(S1), S119–S136.
- Teräsvirta, T., D. Tjøstheim, and C. W. J. Granger (2010). *Modelling Nonlinear Economic Time Series*. Advanced Texts in Econometrics. Oxford University Press.
- Teräsvirta, T. and Y. Yang (2014a). Linearity and Misspecification Tests for Vector Smooth Transition Regression Models. Research Paper 4, CREATES.

- Teräsvirta, T. and Y. Yang (2014b). Specification, Estimation and Evaluation of Vector Smooth Transition Autoregressive Models with Applications. Research Paper 8, CREATES.
- Tong, H. and K. S. Lim (1980). Threshold Autoregression, Limit Cycles and Cyclical Data. *Journal of the Royal Statistical Society. Series B (Methodological)* 42(3), 245–292.
- van Dijk, D. and P. Franses (1999). Modeling Multiple Regimes in the Business Cycle. *Macroeconomic Dynamics* 3(3), 311–340.
- Weise, C. (1999). The Asymmetric Effects of Monetary Policy: A Nonlinear Vector Autoregression Approach. *Journal of Money Credit and Banking* 31(1), 85–108.

Figures

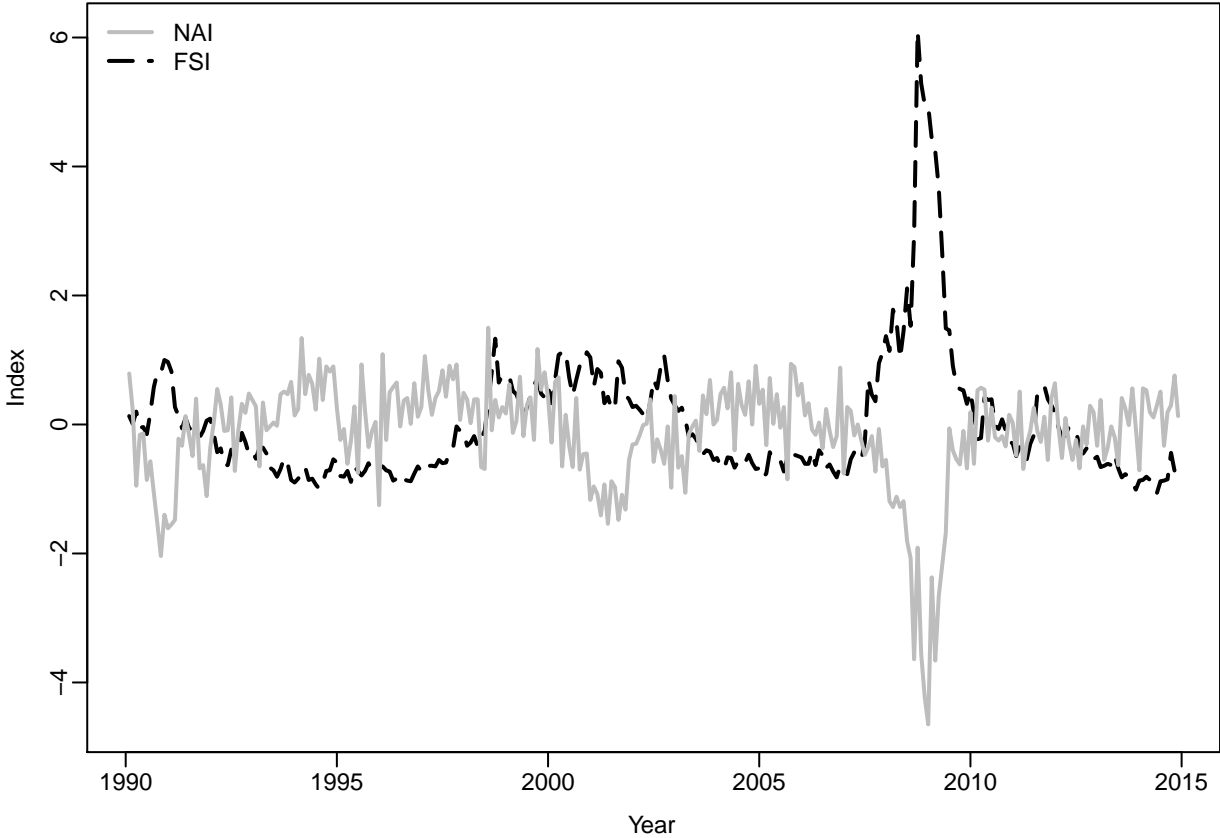


Figure 1: Time series of the Financial Stress and the National Activity Indices.

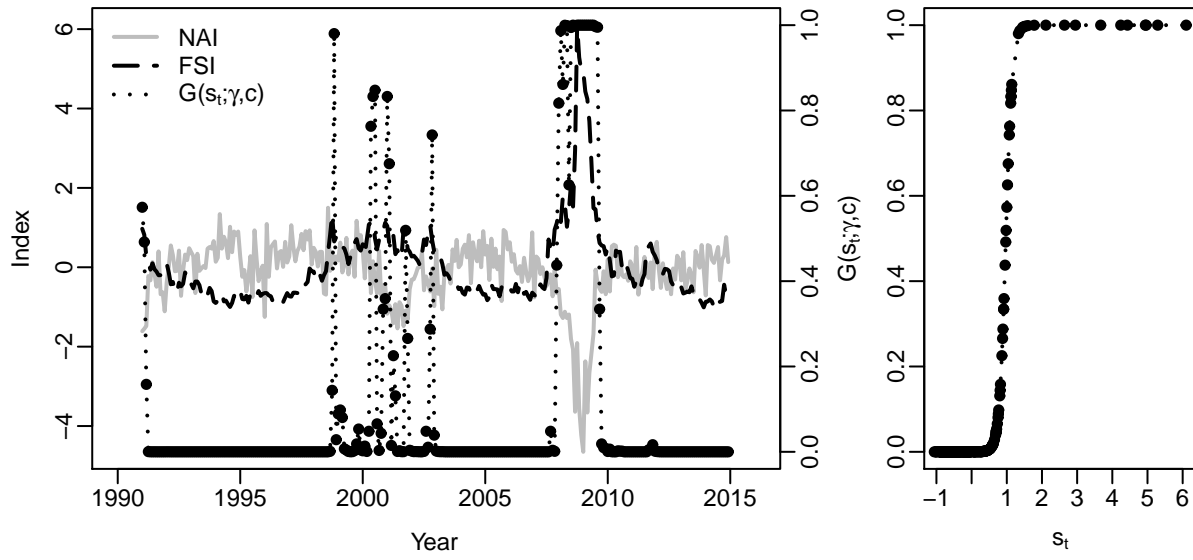


Figure 2: Transition function of the estimated VSTAR, with z_{t-1} as the transition variable.

Note: The right-hand-side panel illustrates transition function in an ascending order of the transition variable, where each dot corresponds to a value of the transition variable, z_{t-1} , on the horizontal axis.

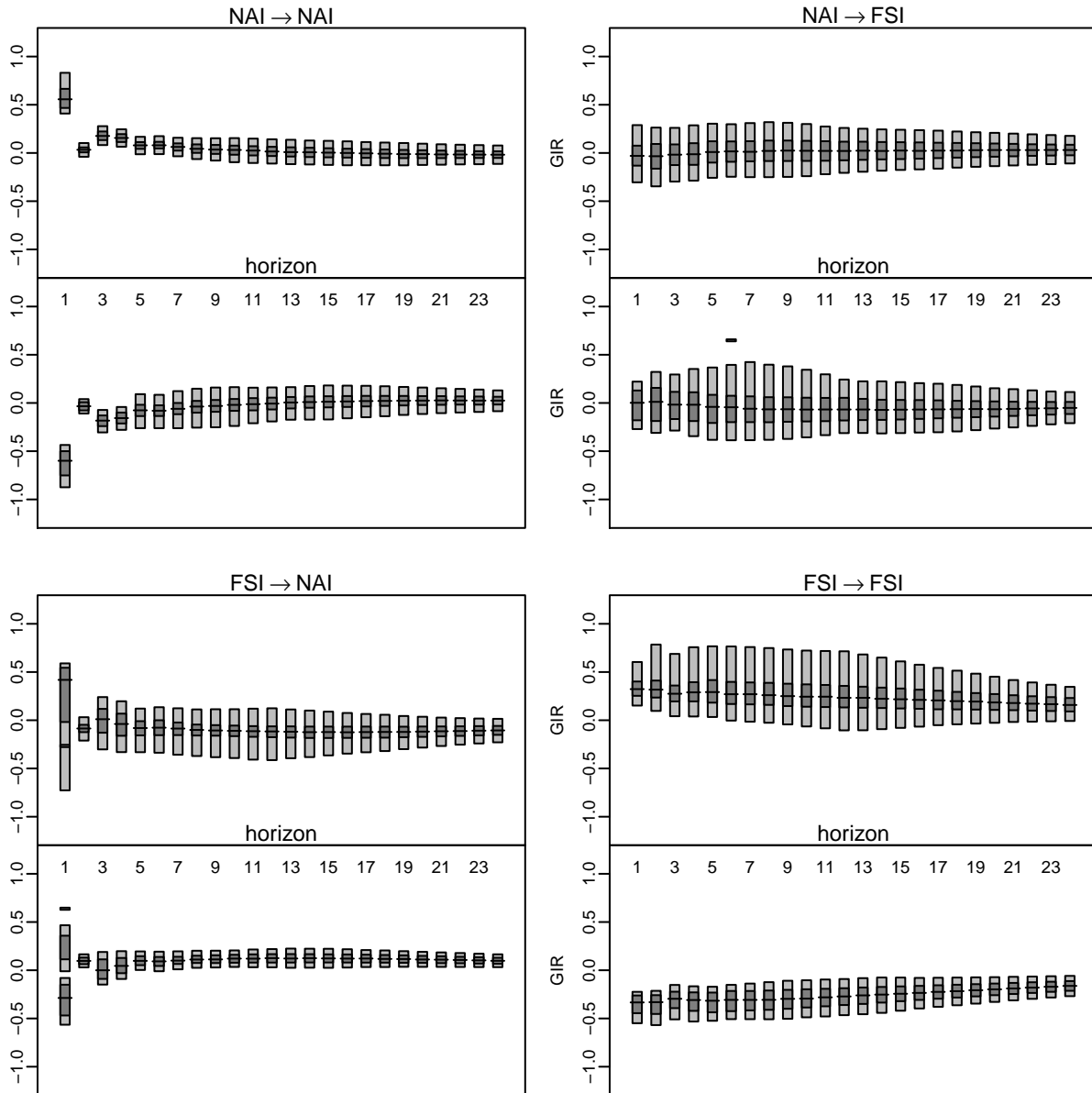


Figure 3: Generalized impulse-responses.

Note: The GIR functions are generated from 100 randomly sampled histories. Bar-plots denote 50 percent (dark shade of gray) and 80 percent (light shade of gray) highest density regions for the GIR densities for $h = 1, \dots, 24$. The top panel features two pairs of plots representing impulse-responses after positive (top) and negative (bottom) NAI shocks. The bottom panel features two pairs of plots representing impulse-responses after positive (top) and negative (bottom) FSI shocks.

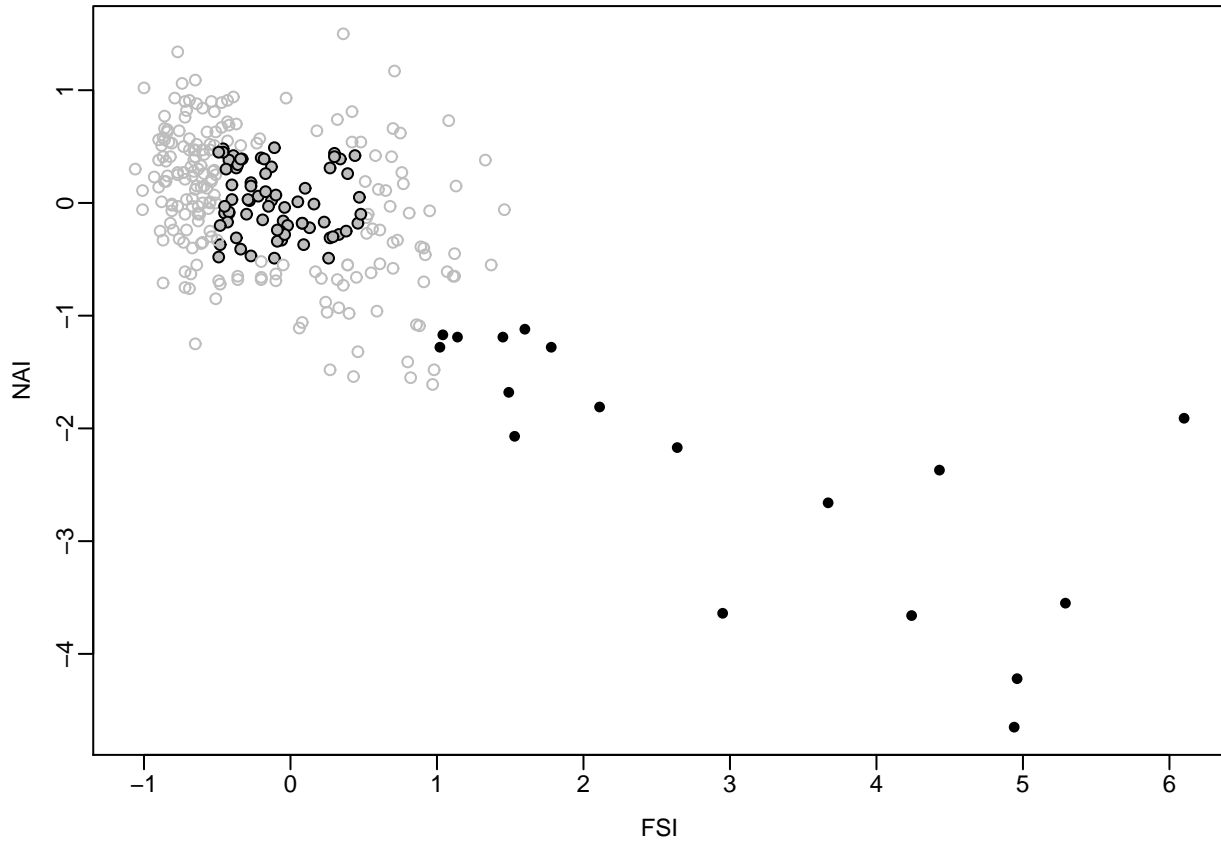


Figure 4: Scatter-plot of the financial stress and the national activity indices.

Note: The black filled dots identify the co-occurrence of the economically and financially “stressed” conditions, while the gray filled dots represent histories associated with the “normal” regime.

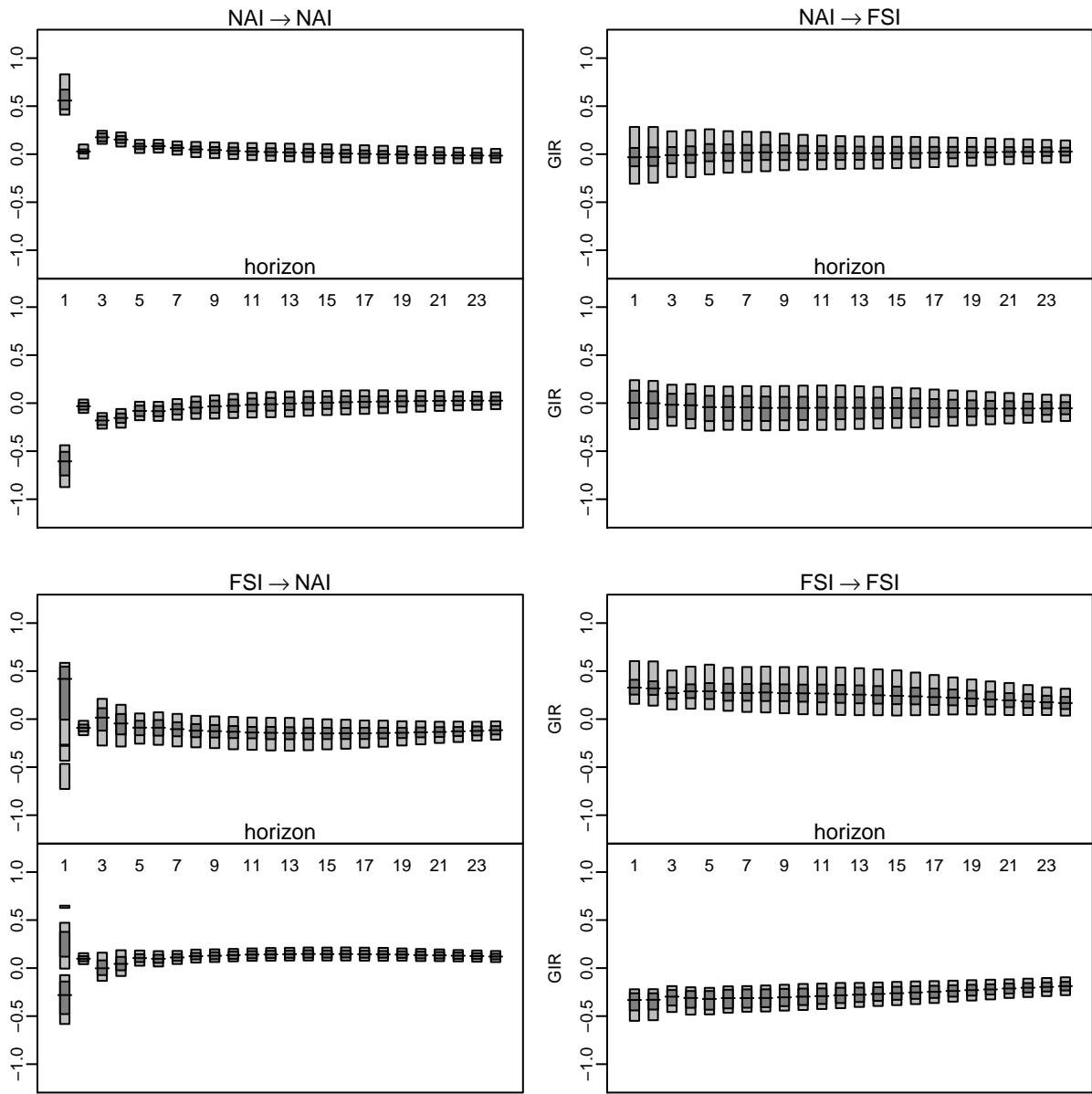


Figure 5: Generalized impulse-responses during normal conditions.

Note: The GIR functions are generated from 18 randomly sampled histories of the normal regime. Bar-plots denote 50 percent (dark shade of gray) and 80 percent (light shade of gray) highest density regions for the GIR densities for $h = 1, \dots, 24$. The top panel features two pairs of plots representing impulse-responses after positive (top) and negative (bottom) NAI shocks. The bottom panel features two pairs of plots representing impulse-responses after positive (top) and negative (bottom) FSI shocks.

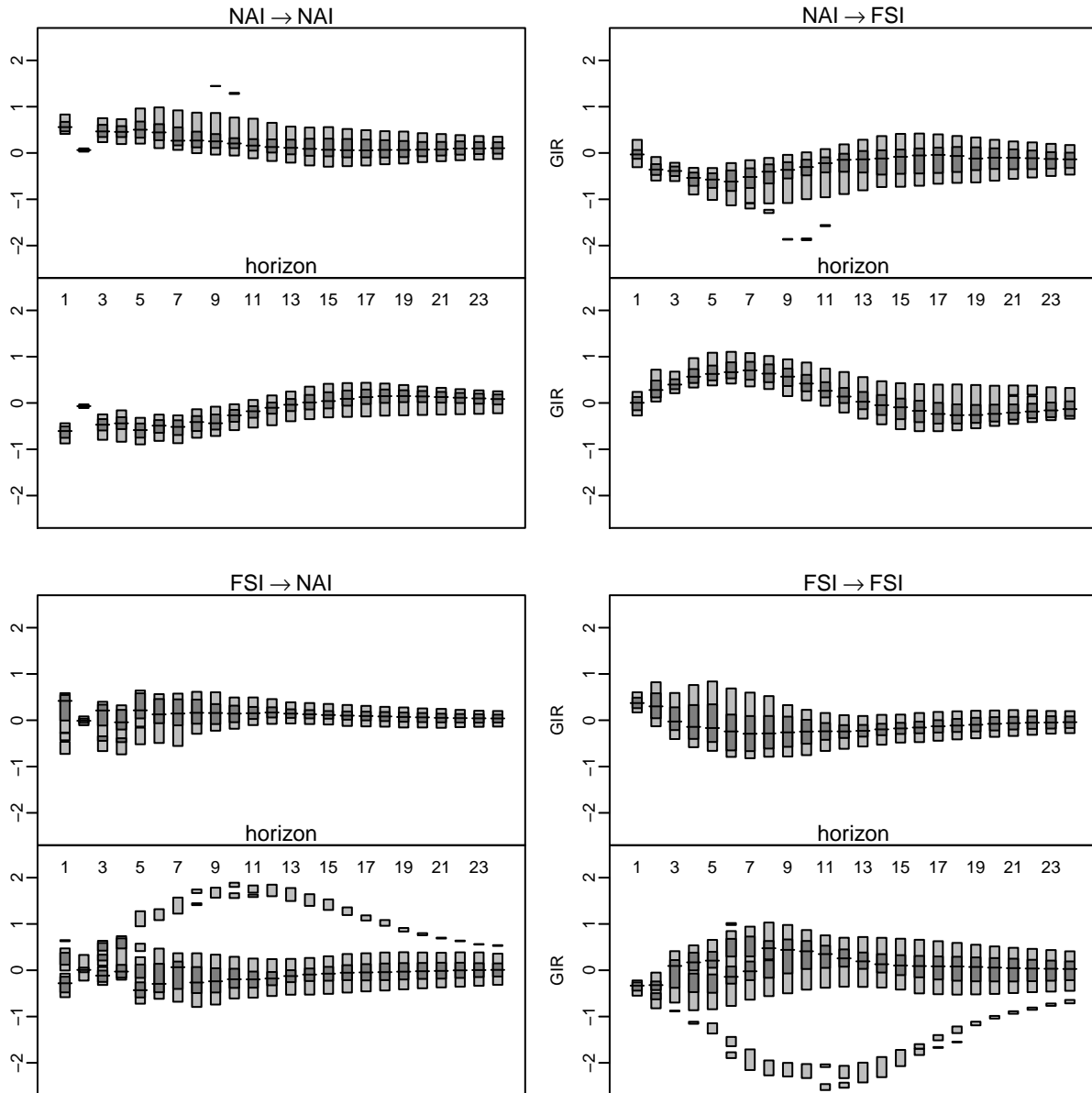


Figure 6: Generalized impulse-responses during stressed conditions.

Note: The GIR functions are generated from 18 histories of the stressed regime. Bar-plots denote 50 percent (dark shade of gray) and 80 percent (light shade of gray) highest density regions for the GIR densities for $h = 1, \dots, 24$. The top panel features two pairs of plots representing impulse-responses after positive (top) and negative (bottom) NAI shocks. The bottom panel features two pairs of plots representing impulse-responses after positive (top) and negative (bottom) FSI shocks.

Tables

Table 1: Linearity and Misspecification Test Results

	LM_R
<i>Test of Linearity</i>	
z_{t-1}	0.001
z_{t-2}	0.005
z_{t-3}	0.002
z_{t-4}	0.024
<i>Test of No Remaining Nonlinearity</i>	
z_{t-1}	0.276
<i>Test of Parameter Constancy</i>	
t^*	0.137
<i>Test of No Serial Correlation</i>	
ac_{t-1}	0.157

Note: The values are probability values of the test statistics; LM_R denotes the Rao's F -test variants of the Lagrange multiplier statistics. z_{t-d} , $d = 1, \dots, 4$, are candidate transition variables, where $\{z_t\}$ represents the financial stress index in period t ; $t^* = t/T$, where T is the total sample size, is used to test for parameter constancy in the estimated VSTAR model, while ac_{t-1} denotes the joint first order serial correlation. See [Teräsvirta and Yang \(2014a,b\)](#) for details.

Table 2: Parameter Estimates

dependent variable	VAR		VSTAR			
	y_t	z_t	y_t		z_t	
			R1	R2	R1	R2
<i>intercept</i>	-0.038 (0.030)	-0.009 (0.018)	0.008 (0.035)	-0.601 (0.197)	-0.010 (0.019)	0.357 (0.109)
y_{t-1}	0.100 (0.058)	-0.122 (0.035)	0.042 (0.065)	0.053 (0.161)	0.006 (0.037)	-0.513 (0.091)
y_{t-2}	0.307 (0.059)	0.011 (0.035)	0.263 (0.064)	0.472 (0.190)	0.015 (0.036)	-0.167 (0.107)
y_{t-3}	0.221 (0.059)	-0.013 (0.036)	0.204 (0.064)	0.216 (0.169)	0.021 (0.036)	-0.286 (0.095)
y_{t-4}	0.022 (0.056)	0.071 (0.034)	0.022 (0.062)	-0.021 (0.151)	0.032 (0.035)	0.049 (0.085)
z_{t-1}	-0.364 (0.095)	1.115 (0.058)	-0.237 (0.165)	0.218 (0.227)	0.990 (0.093)	-0.116 (0.127)
z_{t-2}	-0.093 (0.141)	-0.366 (0.085)	0.167 (0.220)	-0.420 (0.302)	-0.151 (0.124)	-0.422 (0.169)
z_{t-3}	-0.117 (0.142)	0.405 (0.086)	-0.041 (0.221)	-0.126 (0.324)	0.211 (0.124)	0.163 (0.182)
z_{t-4}	0.391 (0.099)	-0.249 (0.060)	-0.038 (0.154)	0.922 (0.239)	-0.080 (0.086)	-0.403 (0.134)

Note: Values are parameter estimates, with standard errors in parentheses underneath. R1 and R2 denote regimes 1 and 2, respectively, with parameters representing the elements of vectors α_0 and α_1 , and matrices B_{0i} and B_{1i} . As such, the dynamics in the first regime are defined by α_0 and B_{0i} , while the dynamics in the second regime are defined by $\alpha_0 + \alpha_1$ and $B_{0i} + B_{1i}$.

Table 3: Forecast Evaluation and Out-of-Sample Granger Causality

	AR	STAR	VAR	VSTAR
$\overline{\text{AIC}}$	4.244	4.150	4.037	4.012
$\hat{\delta}_i$	0.437	0.416	0.421	0.407
$\hat{\delta}_i/\hat{\delta}_{\text{AR}}$		0.952 ^{*†}	0.963 [†]	0.932 ^{*†}
$\hat{\delta}_i/\hat{\delta}_{\text{STAR}}$				0.979
$\hat{\delta}_i/\hat{\delta}_{\text{VAR}}$				0.968 ^{*†}

Note: $\overline{\text{AIC}}$ denotes the average AIC across rolling windows; the AIC measures are calculated as $\ln(SSR) + 2k/R$, where SSR is sum of squared residuals, k is total number of estimated parameters, and R is the length of time series used in estimation. Further, $\hat{\delta}_i$, $i = \{\text{AR}, \text{STAR}, \text{VAR}, \text{VSTAR}\}$, denotes the root mean squared forecast error; $\hat{\delta}_i/\hat{\delta}_j$, $j = \{\text{AR}, \text{STAR}, \text{VAR}\}$, are the ratios of the root mean squared errors from the competing models, i.e., the relative accuracy measures. Finally, ^{*} and [†] denote statistical significance at $\alpha = 0.05$ level, respectively based on the DM test statistic and [McCracken's](#) critical values in the former case, and the CW test statistic and standard normal critical values in the latter case.